LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2018**

16/17/18PST1MC01 / ST 1820 / ST 1815– ADVANCED DISTRIBUTION THEORY

Date: 25-10-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION – A

Answer ALL the questions **(10 x 2 = 20 Marks)**

1. The MGF of a random variable X is given by $\left(\frac{1}{3}+\frac{2}{3}e^t\right)^5$. Obtain $P[X \geq 1]$.
2. Consider the distribution F(x) of a random variable X, $F(x)=\begin{cases} 0 & x<0 \\ \frac{x}{2} & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$. Obtain Find $P[-\frac{1}{2} < X \leq \frac{1}{2}]$ and $P[X = 1]$.
3. Write the density function of a truncated Binomial truncated at 0 and n.
4. Let $X_1, X_2,\ldots, X_n$ be a random sample from Geometric distribution. Show that first order statistic also has Geometric distribution.
5. State Cochran's theorem on Quadratic forms.
6. (i) If the random variable T has t-distribution with n df what is the distribution of T^2 ?

   (ii) If the random variable F has F-distribution with (n_1, n_2)df what is the distribution of 1/F?
7. Let $Q = X'AX$ on the random variables $X_1, X_2,\ldots, X_n$. Obtain E[Q].
8. Let $X_1, X_2,\ldots, X_n$ be iid $N(\mu,\sigma^2)$. Write the distribution of $\overline{X}$ and $(n-1)S^2/\sigma^2$.
9. Write the PGF of Bivariate Binominal distribution. Obtain the marginal distribution of X_1.
10. Define non-central Chi-Square distribution.

SECTION – B

Answer any FIVE questions **(5 x 8 = 40 Marks)**

11. Derive the mean and variance of truncated Poisson distribution truncated at 0.
12. For the distribution function $F(x)=\begin{cases} 0 & x<2 \\ \frac{2}{3}x-1 & 2 \leq x \leq 3 \\ 1 & 3 \leq x \end{cases}$

    Obtain the decomposition of F. Find the mean and variance.
13. Let $X_1, X_2,\ldots, X_n$ be iid random variables such that $X_i \sim N(\mu_i,\sigma^2)$, $i=1,2,\ldots,n$ then show that $\sum_{i=1}^{n} a_i X_i$ and $\sum_{i=1}^{n} b_i X_i$ are independent iff $\sum_{i=1}^{n} a_i b_i = 0$.

14. Let $X_1, X_2, ..., X_n$ be a random sample from $f(x) = \alpha e^{-\alpha x}$, $x > 0, \alpha > 0$. Let $D_{ik} = (n-k+1)[X_{(k)} - X_{(k-1)}]$ where $X_{(k)}$ denotes the k[th] order statistic. Show that $D_1, D_2, ..., D_n$ are iid with pdf f(x).
15. Explain spectral decomposition of matrices. Also prove that the characteristic roots of an idempotent matrix is either 0 or 1 and tr(A)=rank(A).
16. Show that mean>median>mode for lognormal distribution.
17. Explain compound distributions. Let random variable X be such that the pdf $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0,1,2,...$ and the pdf of λ be a gamma distribution $G(\alpha, P)$. Obtain the compound distribution of X.
18. Obtain the MGF of trinomial distribution. Obtain the marginal. Also obtain the correlation between the variables.

SECTION – C

Answer any TWO questions **(2 x 20 = 40 Marks)**

19. a) Show that the lack of memory property characterizes the Geometric distribution.

(10)

b) Show that Binominal distribution is a power series distribution. Obtain the MGF of a power series distribution. Hence obtain the recurrence relation for the cumulants. **(2+2+6)**

20. a) Derive the pdf of non-central t-distribution.

b) Let X_1 and X_2 be independent Gamma random variables such that $X_1 \sim G(\alpha, P_1)$ and $X_2 \sim G(\alpha, P_2)$. Obtain the distribution of $\frac{X_1}{X_1 + X_2}$. **(12+8)**

21. a) Derive the marginal and conditional distributions of Bivariate normal distribution.

b) Derive the MGF of Bivariate normal distribution. Show that X and Y are independent when $\rho = 0$. **(8+12)**

22. a) Derive the PGF of Bivariate Poisson distribution. Hence prove the additive property.

b) If (X_1,X_2) has Bivariate Poisson distribution then obtain the correlation between X_1 and X_2.

c) Let (X_1,X_2) has Bivariate Poisson distribution. Drive the condition for independence of X_1 and X_2. **(8+8+4)**

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